Exercise 25

Calculate y'.

$$\sin(xy) = x^2 - y$$

Solution

Take the derivative of both sides with respect to x.

$$\frac{d}{dx}\sin(xy) = \frac{d}{dx}(x^2 - y)$$

$$\cos(xy) \cdot \frac{d}{dx}(xy) = \frac{d}{dx}(x^2) - \frac{d}{dx}(y)$$

$$\cos(xy) \cdot \left\{ \left[\frac{d}{dx}(x)\right] y + x \left[\frac{d}{dx}(y)\right] \right\} = 2x - \frac{dy}{dx}$$

$$\cos(xy) \cdot \left[(1)y + x\left(\frac{dy}{dx}\right)\right] = 2x - \frac{dy}{dx}$$

$$\cos(xy)\left(y + x\frac{dy}{dx}\right) = 2x - \frac{dy}{dx}$$

Solve for dy/dx.

$$y\cos(xy) + x\cos(xy)\frac{dy}{dx} = 2x - \frac{dy}{dx}$$

$$x\cos(xy)\frac{dy}{dx} + \frac{dy}{dx} = 2x - y\cos(xy)$$

$$[x\cos(xy) + 1]\frac{dy}{dx} = 2x - y\cos(xy)$$

Therefore,

$$\frac{dy}{dx} = \frac{2x - y\cos(xy)}{x\cos(xy) + 1}.$$